INVESTIGATION AND ANALYSIS OF THREE-PHASE INDUCTION MOTOR PERFORMANCE WITH NON-SINUSOIDAL VOLTAGE SUPPLY

by

Richard Paul Wells

UNITED STATES NAVAL POSTGRADUATE SCHOOL

THESIS

INVESTIGATION AND ANALYSIS
OF THREE-PHASE INDUCTION
MOTOR PERFORMANCE WITH
NON-SINUSOIDAL VOLTAGE SUPPLY

by

Richard Paul Wells

September 1968

[redacted]

INVESTIGATION AND ANALYSIS OF THREE-PHASE INDUCTION MOTOR PERFORMANCE WITH NON-SINUSOIDAL VOLTAGE SUPPLY

by

Richard Paul Wells
Lieutenant, United States Navy
B.S., Naval Academy, 1961

[redacted]

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1968

ABSTRACT

The development of vehicles powered by direct-current sources together with the development of the silicon-controlled rectifier has led to the use of squirrel-cage induction motors operating on non-sinusoidal, variable-frequency voltage supplies. A digital computer simulation of the transient and steady-state performance of a three-phase motor is derived and its use in predicting motor operation is illustrated.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS
(Continued)

TABLE OF SYMBOLS AND ABBREVIATIONS

(All rotor parameters referred to the stator)

v	=	Instantaneous value of voltage, volts
i	=	Instantaneous value of current, amperes
L_{ll}	=	Stator phase self-inductance, henry (assumed identical for all three phases)
L_{xx}	=	Rotor phase self-inductance, henry (assumed identical for all three phases)
L_{yx}	=	Stator-rotor phase mutual inductance maximum, henry
Θ	=	Rotor position angle, radians
r	=	Rotor phase resistance ohms
R	=	Stator phase resistance, ohms
T	=	Electromechanical torque developed, newton-meters
T_M	=	Mechanical torque, newton-meters
$p\Theta$	=	Rotor angular velocity, radians/second
J	=	Inertia of system referred to motor, kilogram meters
Z	=	Friction and windage coefficient
Ψ	=	Magnetic flux linkages, weber
p	=	d/dt
A,B,C	=	Stator phases
a,b,c	=	Rotor phases
t	=	Time, seconds
Δt	=	Time increment, seconds

1. INTRODUCTION

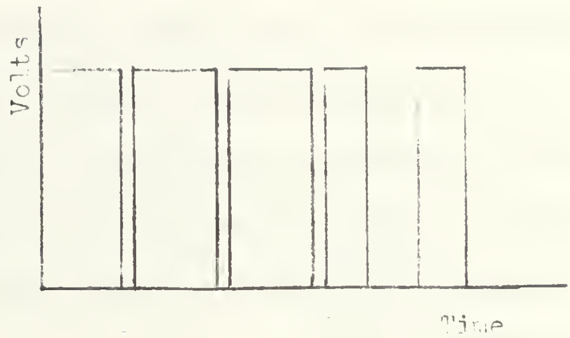
Engineers have long sought a method of combining the simplicity, compactness, low cost, ruggedness and low maintenance characteristics of the squirrel-cage rotor type of induction motor with the speed control characteristics of direct-current motors, involving comutators and brushes, or of wound-rotor induction motors also using brushes.

In recent years two developments have given importance and feasibility to this project.

1) The present and projected development of space vehicles, deep-submergence-oceanographic vessels, and nuclear reactor coolant pumps has demonstrated and intensified the requirement for reliable, variable-speed drives which require no maintenance, not even brush replacement, during their expected lifetime, and which are very reliable. Preferably these drives should be able to operate from a direct-current source such as a fuel cell or battery pack.

2) The development of the silicon-controlled rectifier and its use in bridge-inverters controlled by solid-state logic devices provides a reliable, practical method of obtaining a variable-frequency three-phase power supply from a direct-current source. These inverters commonly contain no moving parts, have very short switching times and appear to be very reliable.

Many descriptions of solid-state inverters have been published in recent years [9] and it will be assumed that voltage wave shapes of the following type are readily available.



Power Conditioner Voltage Output

Figure 1-1

2. ASSUMPTIONS

1) The wave shapes of all the three phase voltages are cyclic and have the same shape.

2) Iron losses are assumed to be small and are not accounted for. Saturation of the magnetic path is neglected.

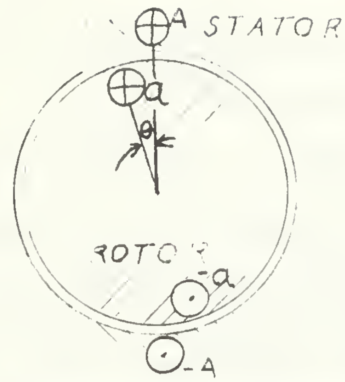
3) Deep-bar effects are not considered.

4) Since the source voltage is applied to one phase in series with two phases in parallel (Y connection), and the windings are assumed to be symmetrical, the phase voltage division is one-third and two-thirds of the source voltage.

5) The rotor is cylindrical as is the inside of the stator. The air gap is therefore uniform.

6) The rotor is of the symmetrical, three-phase, coil-wound type. This is done to facilitate analysis although the basic phenomena occurring in a squirrel-cage rotor are the same.

3. PRINCIPLES OF OPERATION



Single-Phase Motor Model

Figure 3-1

If at time zero a positive voltage v is applied to stator phase A as shown, current i_A will flow as indicated creating flux which will link the turns of phase a. As i_A increases an induced current will be generated in rotor phase a which will oppose the time rate of change of i_A. Fleming's right-hand rule may be used to establish the polarity of this induced current as negative, i.e., in at the bottom and out at the top. By Lenz's law a force or torque is created which acts to turn the rotor in a counterclockwise direction. Consider the sequence of events: v_A creates an exponentially rising current i_A, i_A produces flux linkage Ψ_a, the time rate of change of which generates voltage v_a, and in turn v creates current i_a.

It is obvious that if periodic wave forms are considered i_a must lag i_A. Also, since v_a is created by the time rate of change of i_A, when i_A begins to decrease in value

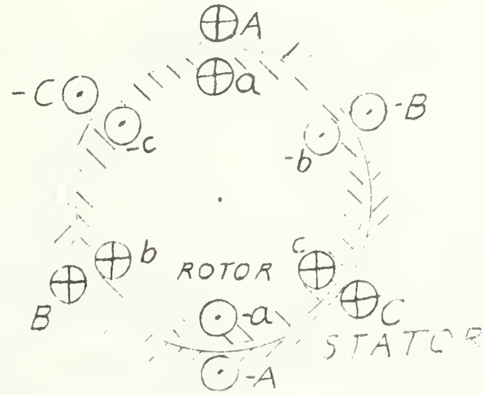
the polarity of induced voltage v_a must reverse. Therefore, at this point it may be assumed that the wave form of i_a must lag the wave form of i_A.

If now the rotor is considered free to rotate and it is assumed that counterclockwise torque is desired, the following relationships must be considered. Let Θ, rotor angle, be zero when coil axes A and a are colinear and let counterclockwise be considered positive. Then the mutual inductance will equal $L_{yx} \cos \Theta$ by definition. Torque, T, will then be equal to $i_A \cdot (-i_a) \cdot [\frac{d}{d\Theta} L_{yx} \cos \Theta]$ which equals $-i_A \cdot (-i_a) \cdot [L_{1x} \sin \Theta]$.

For positive torque with Θ greater than zero and less than Π radians, the product $i_A\ i_a$ must be negative. For Θ between Π and 2Π radians $i_A\ i_a$ must be positive.

Another effect which must be considered is the rotor phase belt "carrying" its current around with it. In the numerical methods solution to follow it will be assumed that electrical transients are fast enough compared to mechanical transients so that Θ may be considered constant for the very short Δt periods involved. The foregoing does not mean that "speed voltages" or counter-electromotive force is neglected, but only that $p\Theta$ and mutual inductance are held constant for the Δt periods.

4. ANALYSIS OF THREE-PHASE MOTOR OPERATION



Three-Phase Motor Model

Figure 4-1

If each coil is considered to have an axis along which a composite flux is created the total flux linkages along each axis may be computed as follows:

$$\Psi_A = i_A L_{11} + i_B L_{yx} \cos(2\Pi/3) + i_C L_{yx} \cos(-2\Pi/3) + i_a L_{yx} \cos(\Theta) + i_b L_{yx} \cos(\Theta + 2\Pi/3) + i_c L_{yx} \cos(\Theta - 2\Pi/3)$$

$$\Psi_B = i_B L_{11} + i_C L_{yx} \cos(2\Pi/3) + i_A L_{yx} \cos(-2\Pi/3) + i_a L_{yx} \cos(\Theta - 2\Pi/3) + i_b L_{yx} \cos(\Theta) + i_c L_{yx} \cos(\Theta + 2\Pi/3)$$

$$\Psi_C = i_C L_{11} + i_A L_{yx} \cos(2\Pi/3) + i_B L_{yx} \cos(-2\Pi/3) + i_a L_{yx} \cos(\Theta + 2\Pi/3) + i_b L_{yx} \cos(\Theta - 2\Pi/3) + i_c L_{yx} \cos(\Theta)$$

$$\Psi_a = i_a L_{xx} + i_b L_{yx} \cos(2\Pi/3) + i_c L_{yx} \cos(-2\Pi/3 + i_A L_{yx} \cos(-\Theta) + i_B L_{yx} \cos(2\Pi/3 - \Theta) + i_C L_{yx} \cos(-2\Pi/3 - \Theta)$$

$$\Psi_b = i_b L_{xx} + i_a L_{yx} \cos(-2\Pi/3) + i_c L_{yx} \cos(2\Pi/3) + i_A L_{yx} \cos(-2\Pi/3 - \Theta) + i_B L_{yx} \cos(-\Theta) + i_C L_{yx} \cos(2\Pi/3 - \Theta)$$

$$\Psi_c = i_c L_{xx} + i_a L_{yx} \cos(2\Pi/3) + i_b L_{yx} \cos(-2\Pi/3) + i_A L_{yx} \cos(2\Pi/3-\Theta) + i_B L_{yx} \cos(-2\Pi/3-\Theta) + i_C L_{yx} \cos(-\Theta)$$

The voltage equations for the physical machine are defined to be:

$$v = iR + p\Psi$$

i.e.:

$$v_A = i_A R + l_{11}\, pi_A + L_{yx} \cos(2\Pi/3)\, pi_B + L_{yx} \cos(-2\Pi/3)\, pi_C + L_{yx} \cos(\Theta)\, pi_a - i_a L_{yx} \sin(\Theta)\, p\Theta + L_{yx} \cos(\Theta+2\Pi/3)\, pi_b - i_b L_{yx} \sin(\Theta+2\Pi/3)\, p\Theta + L_{yx} \cos(\Theta-2\Pi/3)\, pi_c - i_c L_{yx} \sin(\Theta-2\Pi/3)\, p\Theta$$

$$v_B = i_B R + L_{11} pi_B + L_{yx} \cos(2\Pi/3)\, pi_C + L_{yx} \cos(-2\Pi/3)\, pi_A + L_{yx} \cos(\Theta-2\Pi/3)\, pi_a - i_a L_{yx} \sin(\Theta-2\Pi/3)\, p\Theta + L_{yx} \cos(\Theta)\, pi_b - i_b L_{yx} \sin(\Theta)\, p\Theta + L_{yx} \cos(\Theta+2\Pi/3)\, pi_c - i_c L_{yx} \sin(\Theta+2\Pi/3)\, p\Theta$$

$$v_C = i_C R + L_{11}\, pi_C + L_{yx} \cos(2\Pi/3)\, pi_A + L_{yx} \cos(-2\Pi/3)\, pi_B + L_{yx} \cos(\Theta+2\Pi/3)\, pi_a - i_a L_{yx} \sin(\Theta+2\Pi/3)\, p\Theta + L_{yx} \cos(\Theta-2\Pi/3)\, pi_b - i_b L_{yx} \sin(\Theta-2\Pi/3)\, p\Theta + L_{yx} \cos(\Theta)\, pi_c - i_c L_{yx} \sin(\Theta)\, p\Theta$$

$$v_a = i_a r + L_{xx}\, pi_a + L_{yx} \cos(2\Pi/3)\, pi_b + L_{yx} \cos(-2\Pi/3)\, pi_c + L_{yx} \cos(-\Theta)\, pi_A + i_A L_{yx} \sin(-\Theta)\, p\Theta + L_{yx} \cos(2\Pi/3-\Theta)\, pi_B + i_B L_{yx} \sin(2\Pi/3-\Theta)\, p\Theta + L_{yx} \cos(-2\Pi/3-\Theta)\, pi_C + i_C L_{yx} \sin(-2\Pi/3-\Theta)\, p\Theta$$

$$v_b = i_b r + L_{xx}\, pi_b + L_{yx} \cos(-2\Pi/3)\, pi_a + L_{yx} \cos(2\Pi/3)$$

$$pi_c + L_{yx} \cos(-2\Pi/3-\Theta)\, pi_A + i_A L_{yx} \sin(-2\Pi/3-\Theta)\, p\Theta$$

$$+ L_{yx} \cos(-\Theta)\, pi_B + i_B L_{yx} \sin(-\Theta)\, p\Theta + L_{yx} \cos$$

$$(2\Pi/3-\Theta)\, pi_C + i_C L_{yx} \sin(2\Pi/3-\Theta)\, p\Theta$$

$$v_c = i_c r + L_{xx}\, pi_c + L_{yx} \cos(2\Pi/3)\, pi_a + L_{yx} \cos(-2\Pi/3)$$

$$pi_b + L_{yx} \cos(2\Pi/3-\Theta)\, pi_A + i_A L_{yx} \sin(2\Pi/3-\Theta)\, p\Theta$$

$$+ L_{yx} \cos(-2\Pi/3-\Theta)\, pi_B + i_B L_{yx} \sin(-2\Pi/3-\Theta)\, p\Theta$$

$$+ L_{yx} \cos(-\Theta)\, pi_C + i_C L_{yx} \sin(-\Theta)\, p\Theta$$

If, for simplicity of notation, terms are redefined as follows;

$$D = L_{yx} \cos(2\Pi/3)$$

$$E = L_{yx} \cos(-2\Pi/3)$$

$$F = L_{yx} \cos(2\Pi/3-\Theta)$$

$$G = L_{yx} \cos(2\Pi/3+\Theta)$$

$$H = L_{yx} \cos(-2\Pi/3-\Theta)$$

$$I = L_{yx} \cos(-2\Pi/3+\Theta)$$

$$J = L_{yx} \cos(\Theta)$$

$$K = L_{yx} \cos(-\Theta)$$

$$M = L_{yx} \sin(\Theta)\, p\Theta$$

$$N = L_{yx} \sin(-\Theta)\, p\Theta$$

$$P = L_{yx} \sin(2\Pi/3+\Theta)\, p\Theta$$

$$Q = L_{yx} \sin(2\Pi/3-\Theta)\, p\Theta$$

$$S = L_{yx} \sin(-2\Pi/3+\Theta)\, p\Theta$$

$$T = L_{yx} \sin(-2\Pi/3-\Theta)\, p\Theta$$

The equations in matrix form are:

$$\begin{bmatrix} v_A \\ v_B \\ v_C \\ v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & O & O & -M & -P & -S \\ O & R & O & -S & -M & -P \\ O & O & R & -P & -S & -M \\ N & Q & T & r & O & O \\ T & N & Q & O & r & O \\ Q & T & N & O & O & r \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix}$$

$$+ \begin{bmatrix} L_{11} & D & E & J & G & I \\ E & L_{11} & D & I & J & G \\ D & E & L_{11} & G & I & J \\ K & F & H & L_{xx} & D & E \\ H & K & F & E & L_{xx} & D \\ F & H & K & D & E & L_{xx} \end{bmatrix} \begin{bmatrix} pi_A \\ pi_B \\ pi_C \\ pi_a \\ pi_b \\ pi_c \end{bmatrix}$$

It may be well to note at this point that no assumptions or specifications have been made as to voltage waveform or frequency. It has been assumed that the motor windings are symmetrical, an assumption which simplified the notation and in addition allowed the use of the original voltage distribution assumption for a three-wire system. The modification required to consider a four-wire unbalanced system is obvious and will not be pursued.

The system of nonlinear differential equations obtained may be solved only if Θ and $p\Theta$ may be considered constant. Since the transient solution is desired this can only be justified for very short time periods.

The application of numerical methods and the use of a computer may be expected to produce an inexact but useful solution to the problem.

Let the previously derived system of matrix equations be expressed by:

$$[v] = [H][i] + [K][pi]$$

then:

$$[pi] = [K]^{-1}[v] - [K]^{-1}[H][i] \ .$$

Torque generated in the constant air-gap machine is equal to the summation of the product of stator phase currents, rotor phase currents, and the space rate of change in their respective mutual inductances.

Therefore:

$$\begin{aligned} T = {} & -i_A i_a L_{yx} \sin \Theta - i_A i_b L_{yx} \sin (\Theta+2\Pi/3) - i_A i_c L_{yx} \\ & \sin (\Theta-2\Pi/3) - i_B i_b L_{yx} \sin \Theta - i_B i_c L_{yx} \sin (\Theta+2\Pi/3) \\ & - i_B i_a L_{yx} \sin (\Theta-2\Pi/3) - i_C i_c L_{yx} \sin \Theta - i_C i_a L_{yx} \\ & \sin (\Theta+2\Pi/3) - i_C i_b L_{yx} \sin (\Theta-2\Pi/3) \end{aligned}$$

The mechanical equation for torque:

$$T - T_m = J(p(p\Theta)) + Zp\Theta$$

may be solved for $p(p\Theta)$:

$$p(p\Theta) = (T-T_m-Zp\Theta)/J \ .$$

If load torque is assumed to be known, predictor-corrector methods may be used as follows:

PREDICTOR

$$(pi)_n = f(\Theta,p\Theta,i)_n$$

$$p(p\Theta)_n = f(i,\Theta)_n$$

$$i_{n+1} = i_n + (\Delta t)(pi)_n$$

CORRECTOR

$$(p\underline{i})_n = f(\Theta,p\Theta,i)_{n+1}$$

$$\underline{I}_{n+1} = I_n + dt\ (pI+P\underline{I})/2$$

Check

$$\text{Error} = \frac{\underline{I}_{n+1} - I_{n+1}}{\underline{I}_{n+1}}$$

If the absolute value of error is found to be unacceptable, set $I_{n+1} = \underline{I}_{n+1}$ and recompute $\underline{I}_{n+1}$. If the error values are acceptable, torques, rotor angles and angular velocity may be determined preparatory to the next step. Adjusting Δt after each cycle of computations will permit saving of computer time while maintaining the desired degree of accuracy.

Note that all motor parameters used in determining expected transients or steady-state response may be determined by relatively simple laboratory tests of a given machine.[5] Stator and rotor copper (I^2R) loss calculations may be incorporated into the program.

It is theoretically possible to account for hysteresis and eddy-current losses by detailed analysis of flux changes and the magnetic path. However, the parameters required for

these calculations are not normally available to the engineer for an "off the shelf" motor. Since the induction motor circuits themselves are heavily inductive it may optimistically be assumed that the circuits act as a low-pass filter and typical loss figures for the same class of motor operating on a three-phase balanced sinusoidal voltage supply may be used.

5. APPLICATION

For purposes of illustration a 20-horsepower, 440-volt, three-phase, 60-Hz., 3470-rpm, two-pole, squirrel-cage induction motor is selected. The stator windings are WYE connected and the phase model parameters, as determined by blocked-rotor and no-load tests[5] are as follows:

$R = 0.30$ ohms

$r = 0.36$ ohms

$L_l = L_x = 0.00167$ henrys (leakage inductances)

L_{yx} model $= 0.0723$ henrys (magnetizing inductances)

The model magnetizing inductance being three-halves the actual value, the inductance values used are:

$$L_{ll} = L_l + L_{yx}\ (2/3) = 0.04987$$

$$L_{xx} = L_{ll} = 0.04987$$

$$L_{yx} = L_{yx}\ \text{model}\ (2/3) = 0.0482$$

The mechanical load is arbitrarily assumed to be a propeller with load torque equal to 0.000315 times the square of the angular velocity in radians per second. This coefficient is selected to require 19.5 horsepower delivered to

the load at rated speed and the friction coefficient (0.0282) accounts for the remaining one-half horsepower at rated conditions.

The program following, Appendix I, was successfully run on an IBM 360 computer. Limiting the storage requirements to that required for 200 steps at a time allowed double-precision computation using less than 100,000 bytes of core.

IBM supplied the subroutine for matrix inversion (GAUSS3). Provision has been made to avoid multiplication or division by near-zero or zero values.

SINUSOIDAL VOLTAGE SUPPLY

In order to obtain a standard of comparison, a three-phase, 60-Hz, sinusiodal voltage supply was assumed for simulation purposes. Figures 6-1 through 6-6 display the computed current wave-forms and generated torque. The areas of special interest are considered to be motor starting transients and steady-state performance.

NON-SINUSOIDAL VOLTAGE SUPPLY

A 60-Hz, pulsed voltage supply is assumed as an output from the power conditioner which, in accordance with assumption number four, is resolved in the ratio one-third and two-thirds across the stator phases. Computed phase voltage wave-forms, current wave-forms and generated torque values are plotted against time for starting and steady-state conditions in Figures 6-7 through 6-10. Voltage magnitudes are computed to have the same rms value as the rated sinusoidal voltage.

VARIABLE-FREQUENCY VOLTAGE SUPPLY

Assumptions are the same as above except that tachometer feedback is assumed to control the frequency and thus the pulse widths of the stator voltage supply. If the circuit model Thévenin equivalent is solved as a function of phase-voltage frequency and rotor angular velocity for maximum power transfer to the load, and thus maximum generated torque, and if stator-phase resistance is neglected, the optimum voltage frequency is found to be equal to rotor angular velocity plus a constant. The results of this simulation are recorded in Figures 6-11 through 6-13.

While very large starting torques are obtained, the currents generated are excessive and any motor not especially designed would probably be destroyed by a short period of operation involving a series of mechanical transients. The current requirements and heat dissipation requirements for the power conditioner would also far exceed those required for steady-state operation.

Consider the steady-state relations that magnetic flux linkage per pole is proportional to the ratio of rms voltage to frequency and that torque generated is proportional to the space rate of change of magnetic flux linkages.[5] Consider also that the variable-frequency pulse width described above is one-sixth of the stator-voltage cycle time. The rms voltage for a given frequency may be controlled by varying the pulse width up to a maximum of one-sixth of the cycle time. If the pulse width is made proportional to frequency,

the conditions quoted above are satisfied; therefore, maximum rated torque will be available during mechanical transients, and rms currents will be reduced. The results of this simulation, including the described voltage wave-form, are recorded as Figures 6-14 through 6-17.

6. CONCLUSIONS

Results displayed in Figures 6-1 through 6-13 compare favorably with those obtained by other methods.[8] Figure 6-18 is a comparison of $p\Theta$ vs. time for a given motor operating on different voltage wave-forms.

From the data obtained the following conclusions may be drawn:

The pulsed wave-form output obtainable from a power conditioner is useable for squirrel-cage motor operation.

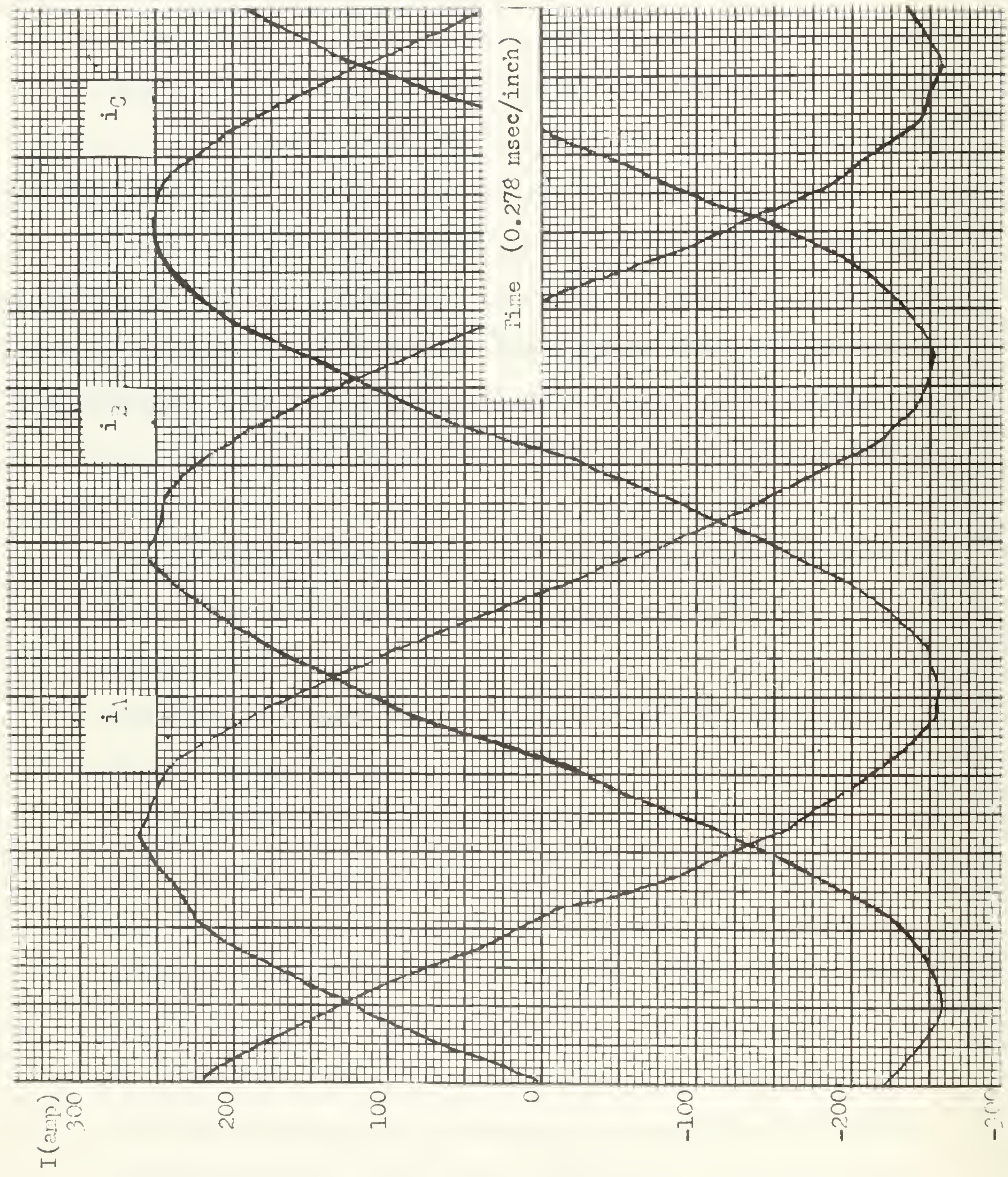
Variable speed control may be obtained by variation of power conditioner frequency.

If the load requirements are such that few large speed changes are expected, such as normal operation of a nuclear reactor coolant pump, the added complexity of the power conditioner required to control pulse-width will probably not be justified.

If large and frequent speed changes are expected, as would probably be the case in propelling an oceanographic vessel, the added complexity involved in pulse-width control is justified.

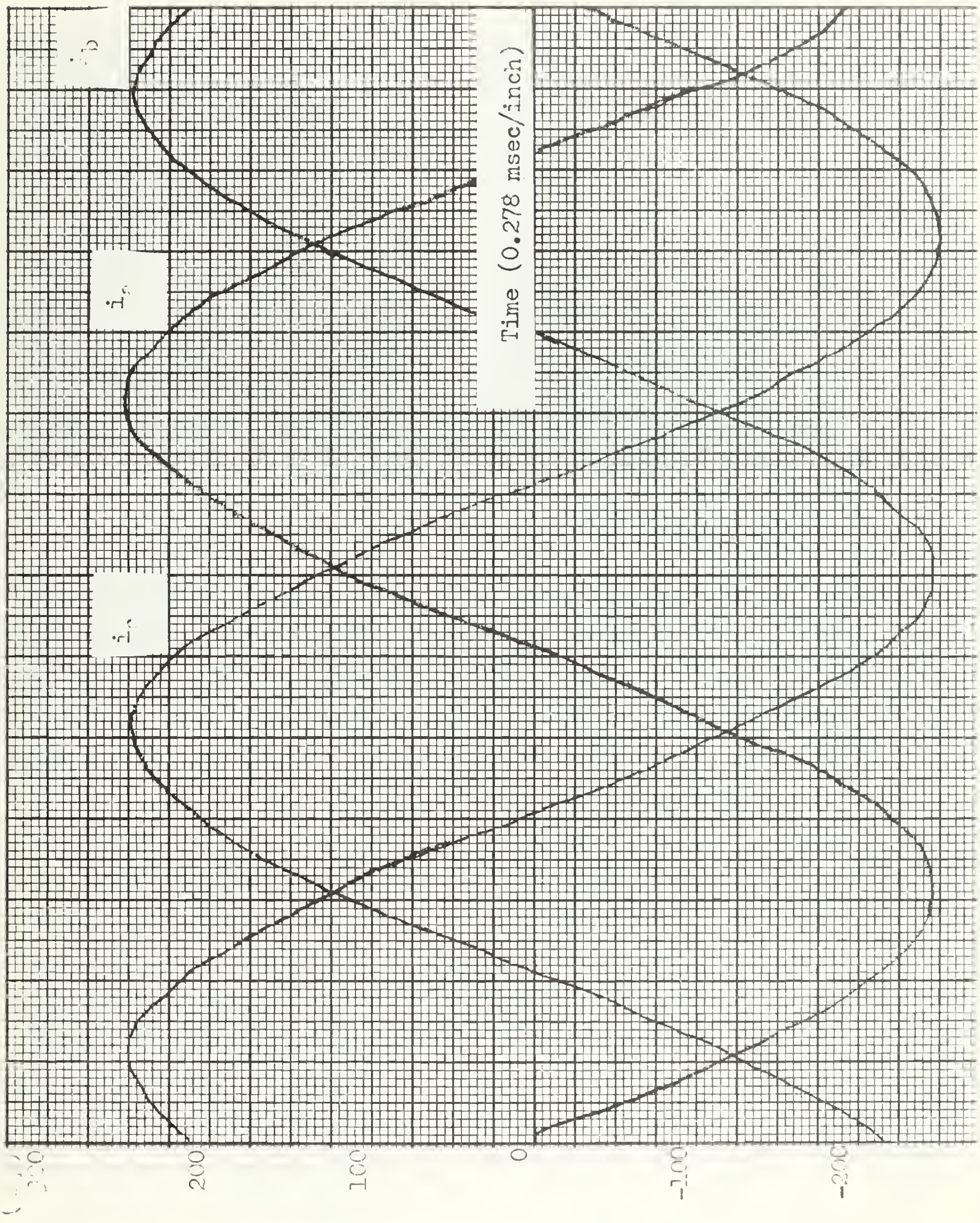
The computer program used to simulate the last wave-form described is presented as Appendix I. Computations using 60 time increments per stator voltage cycle required approximately 11 minutes of IBM 360 Computer time to simulate two and one-half seconds of motor transient time.

Areas of further investigation should include consideration of the effects of magnetic saturation and its effect on current and generated torque. Inclusion of power conditioner operation, copper losses, and vibration calculations in the simulation would improve the program's validity and thus its usefulness.



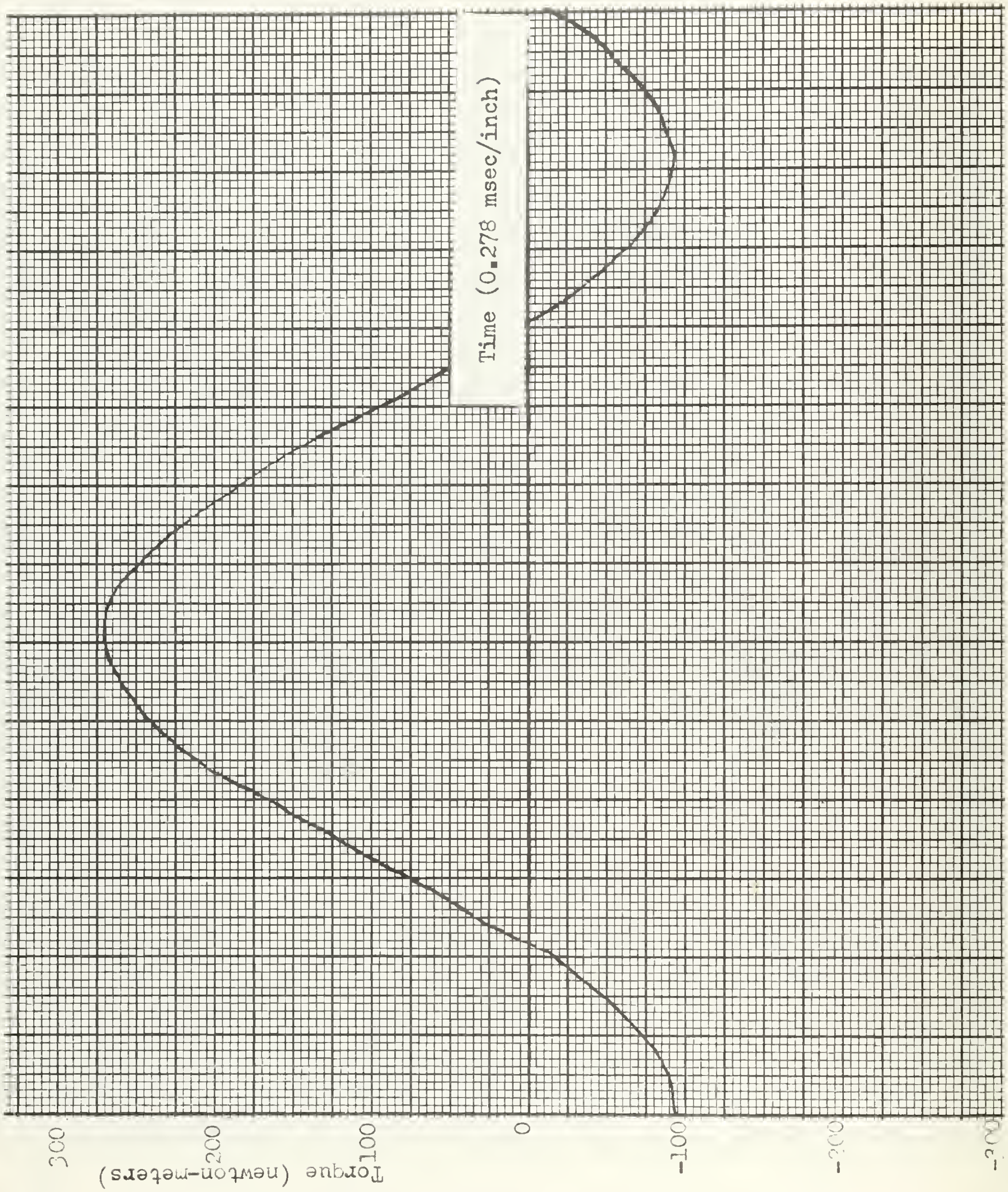
Stator Phase Currents vs. Time. Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-1



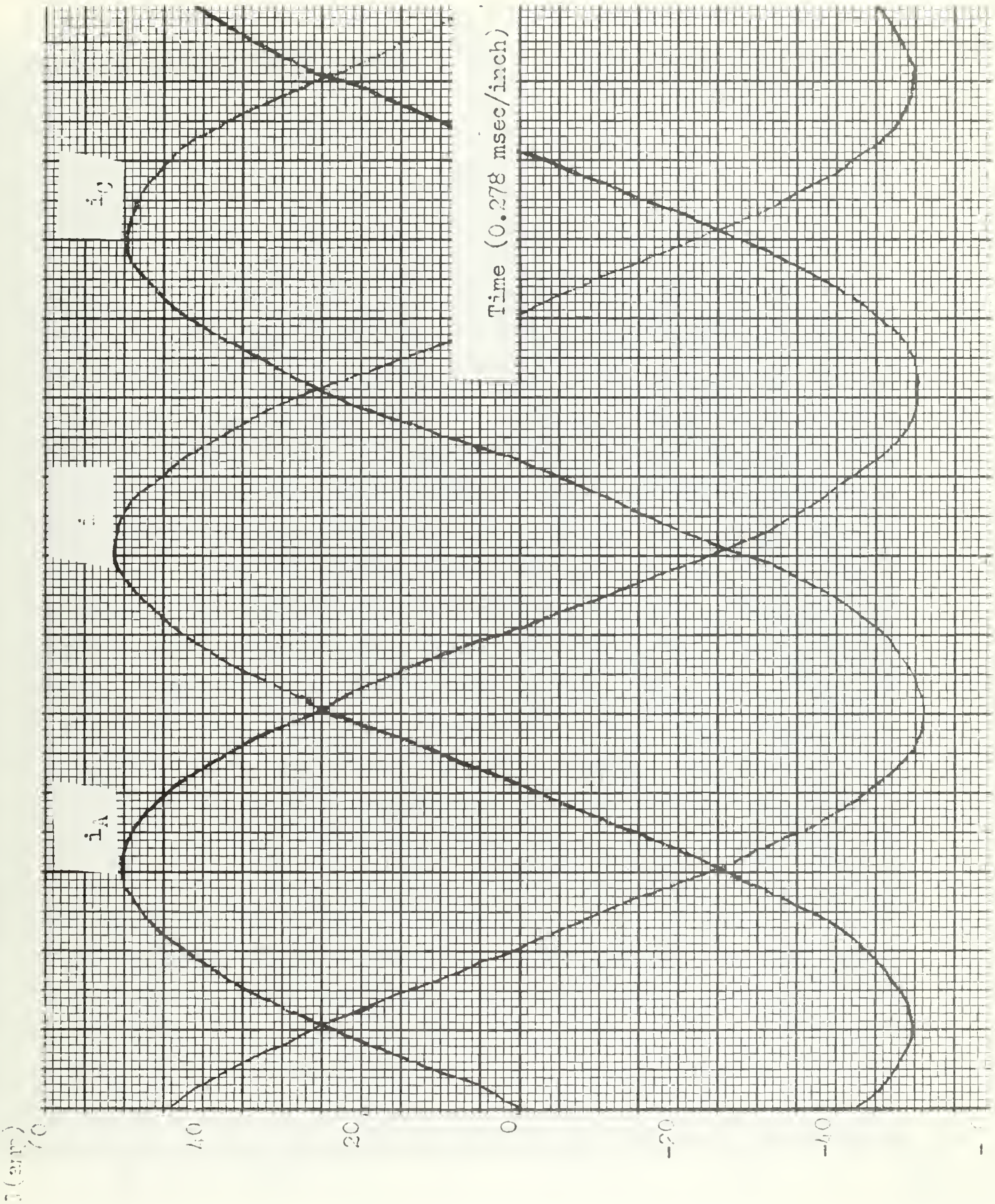
Rotor Phase Currents vs. Time. Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-2



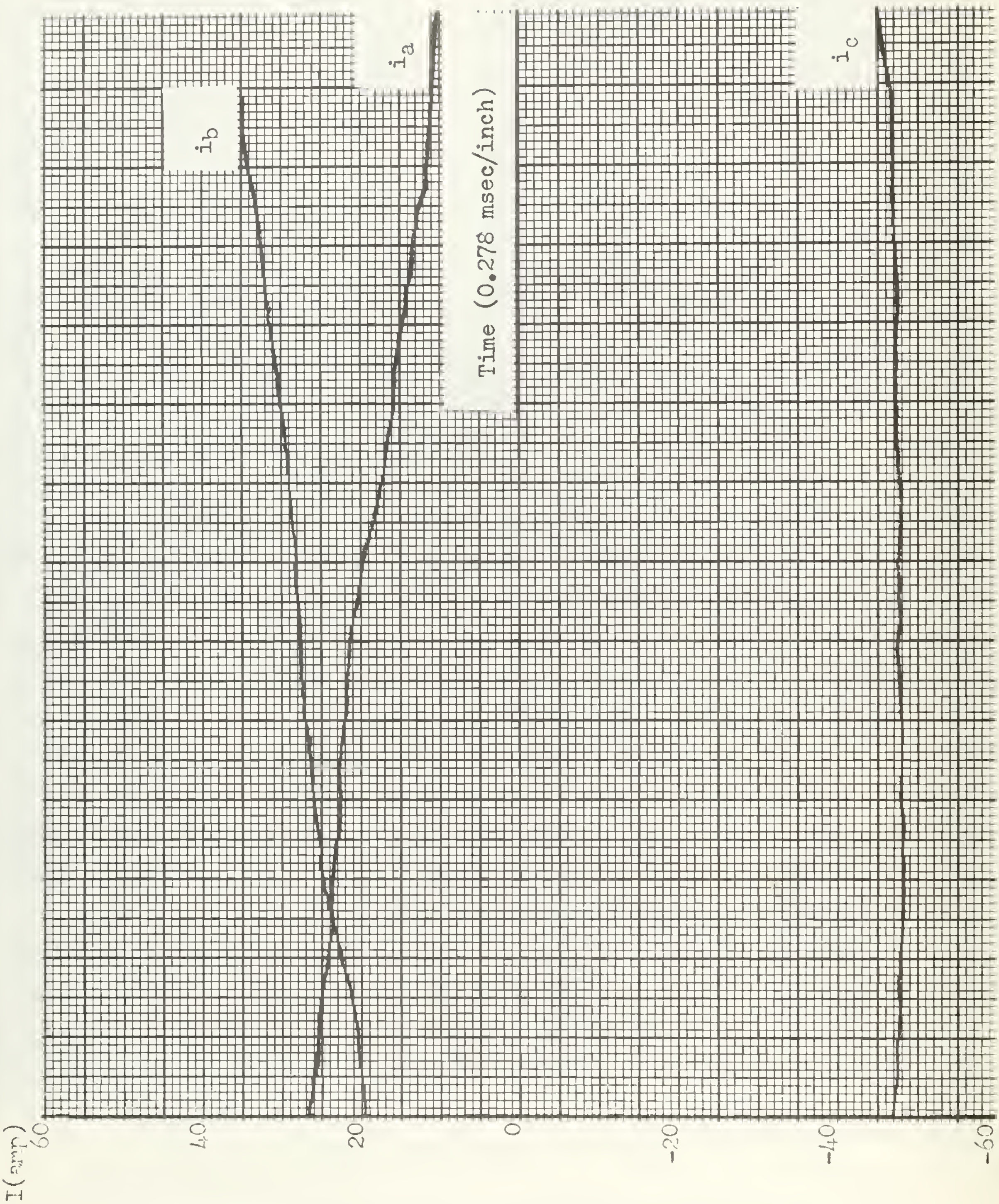
Torque Generated vs. Time. Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-3



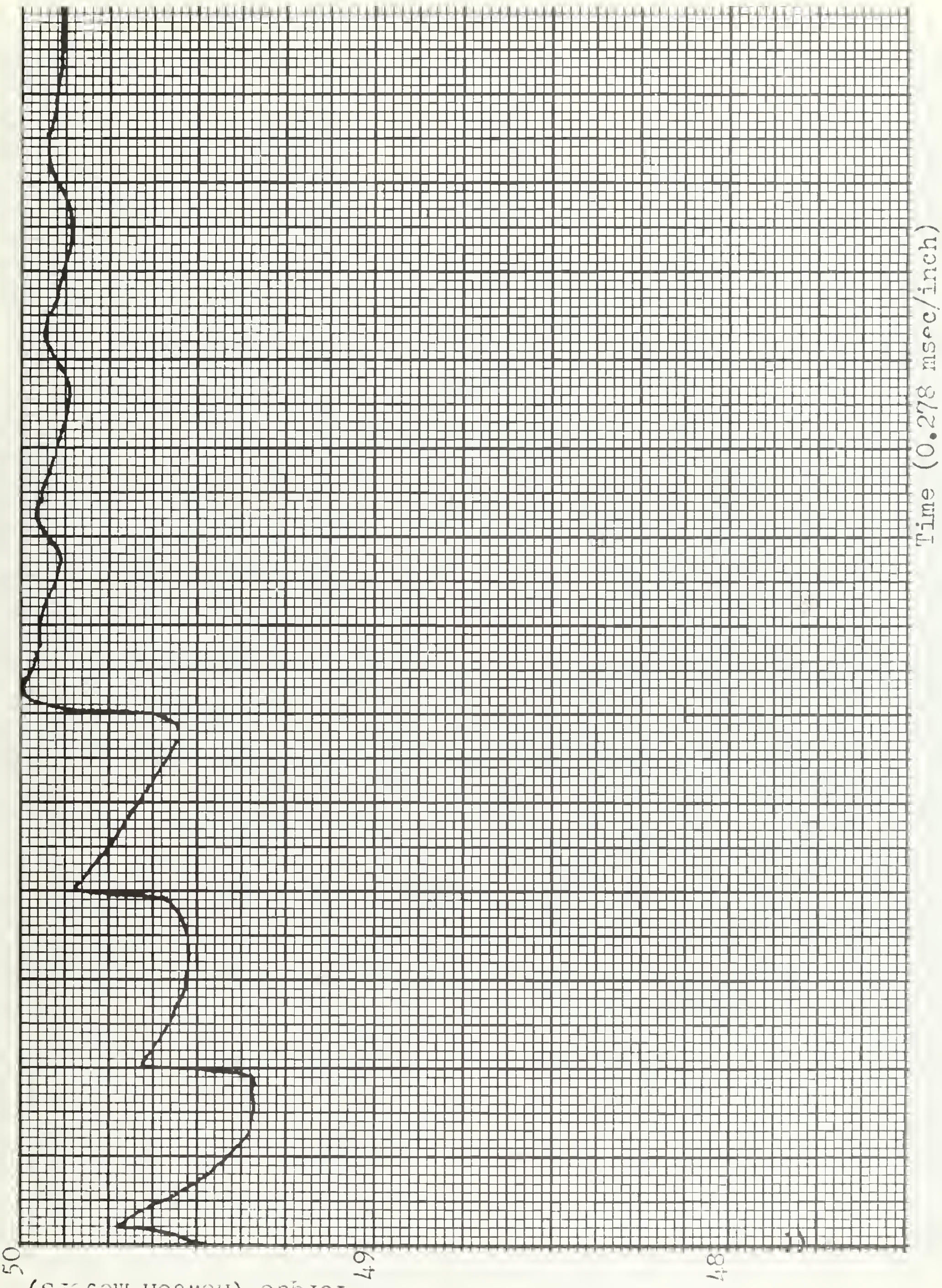
Stator Phase Currents vs. Time. Sinusoidal Voltage Supply. Motor in Steady-State Operation.

Figure 6-4



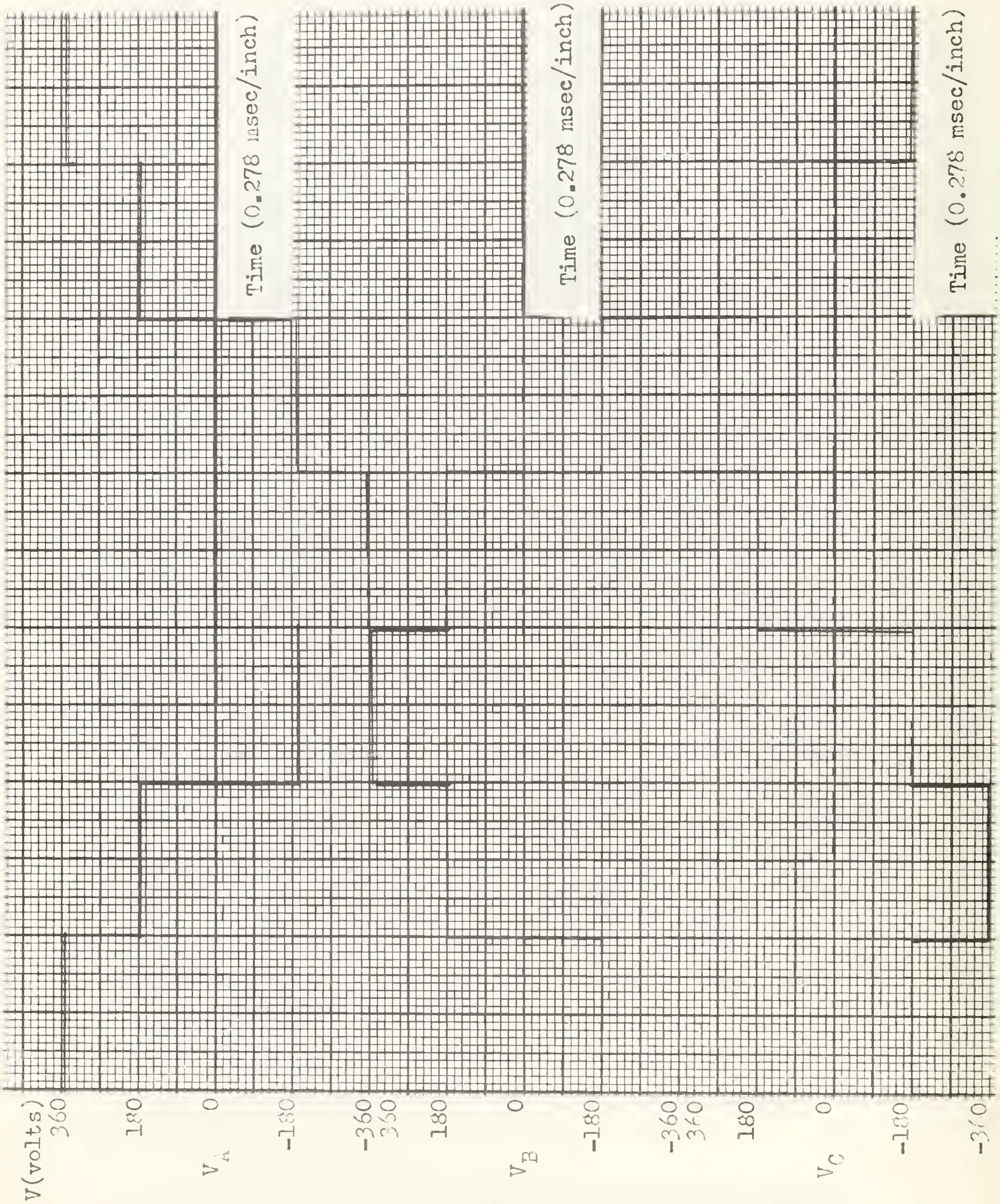
Rotor Phase Currents vs. Time. Sinusoidal Voltage Supply. Motor in Steady-State Operation.

Figure 6-5



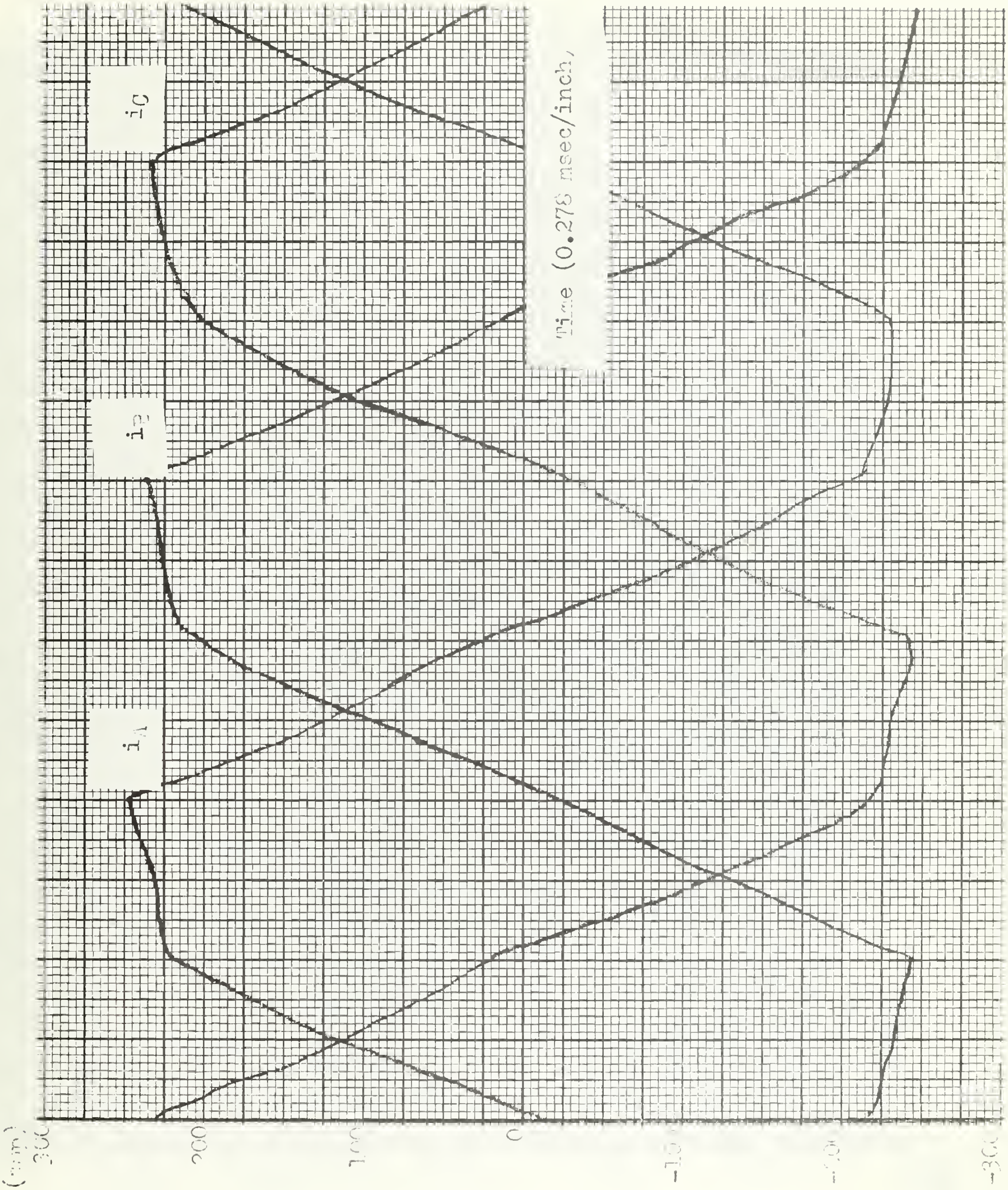
Torque Generated vs. Time. Sinusoidal Voltage Supply. **Motor in Steady-State** Operation.

Figure 6-6



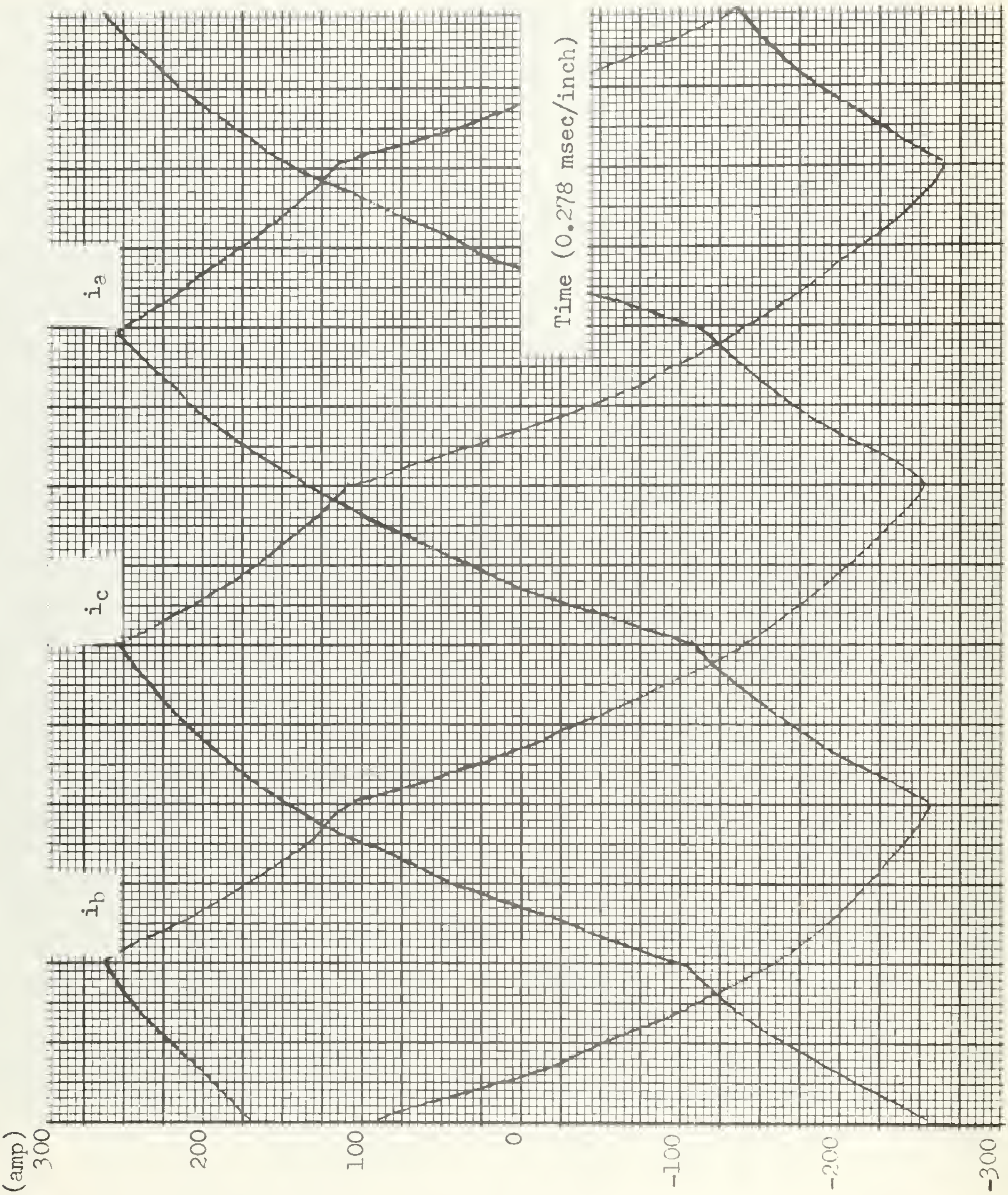
Stator Phase Voltages vs. Time. Constant-Frequency, Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-7



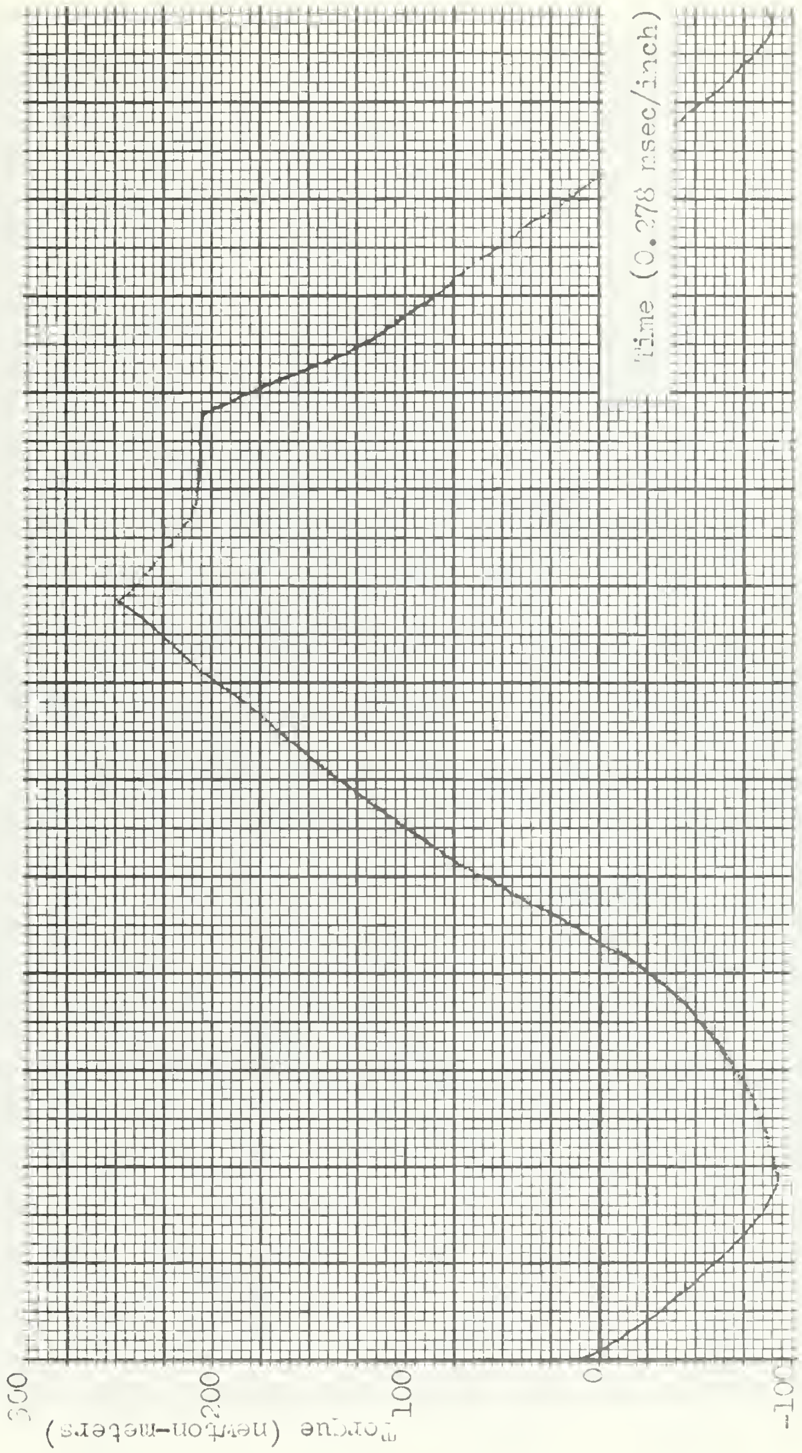
Stator Phase Currents vs. Time. Constant-Frequency, Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-8



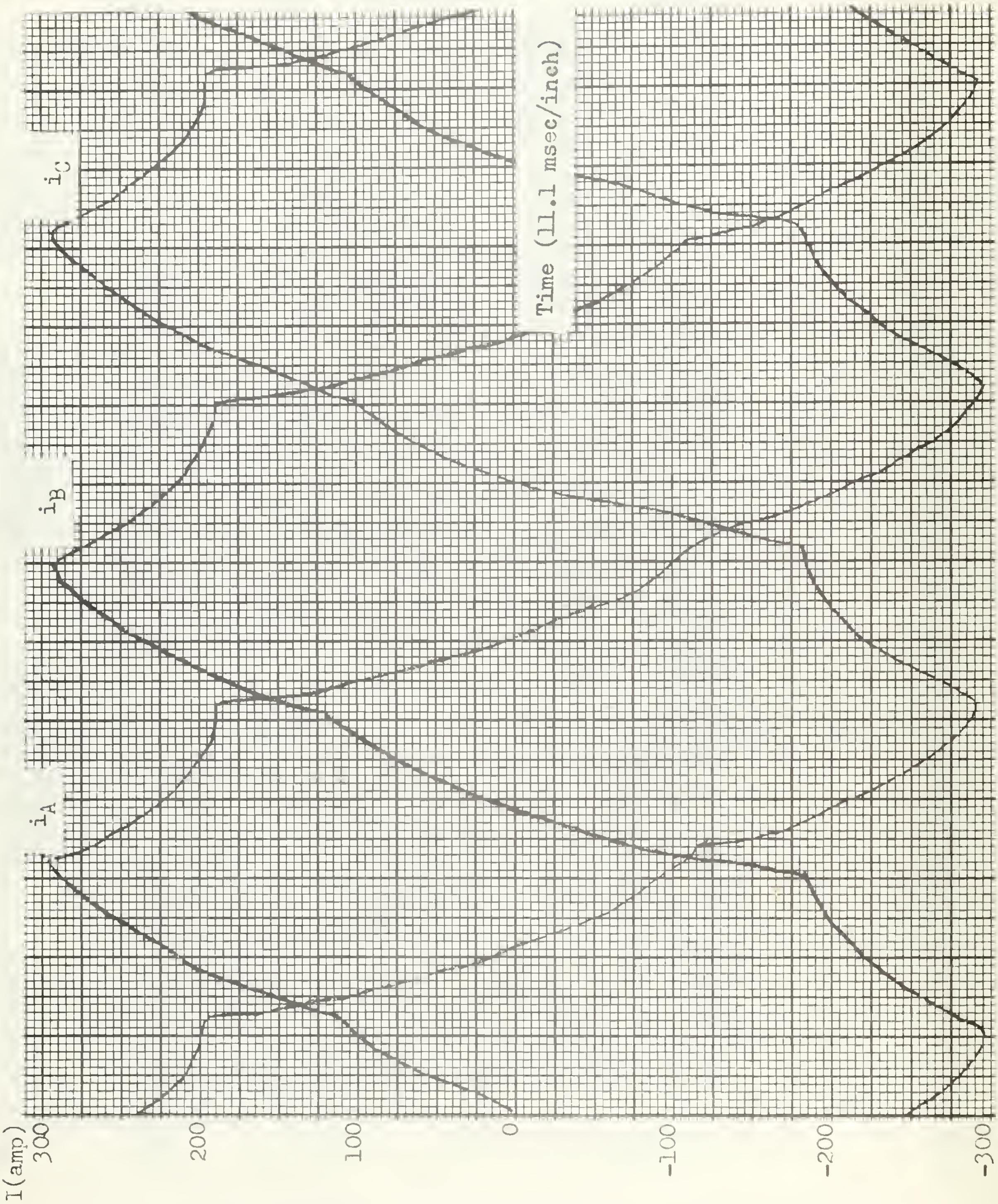
Rotor Phase Currents vs. Time. Constant-Frequency, Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-9



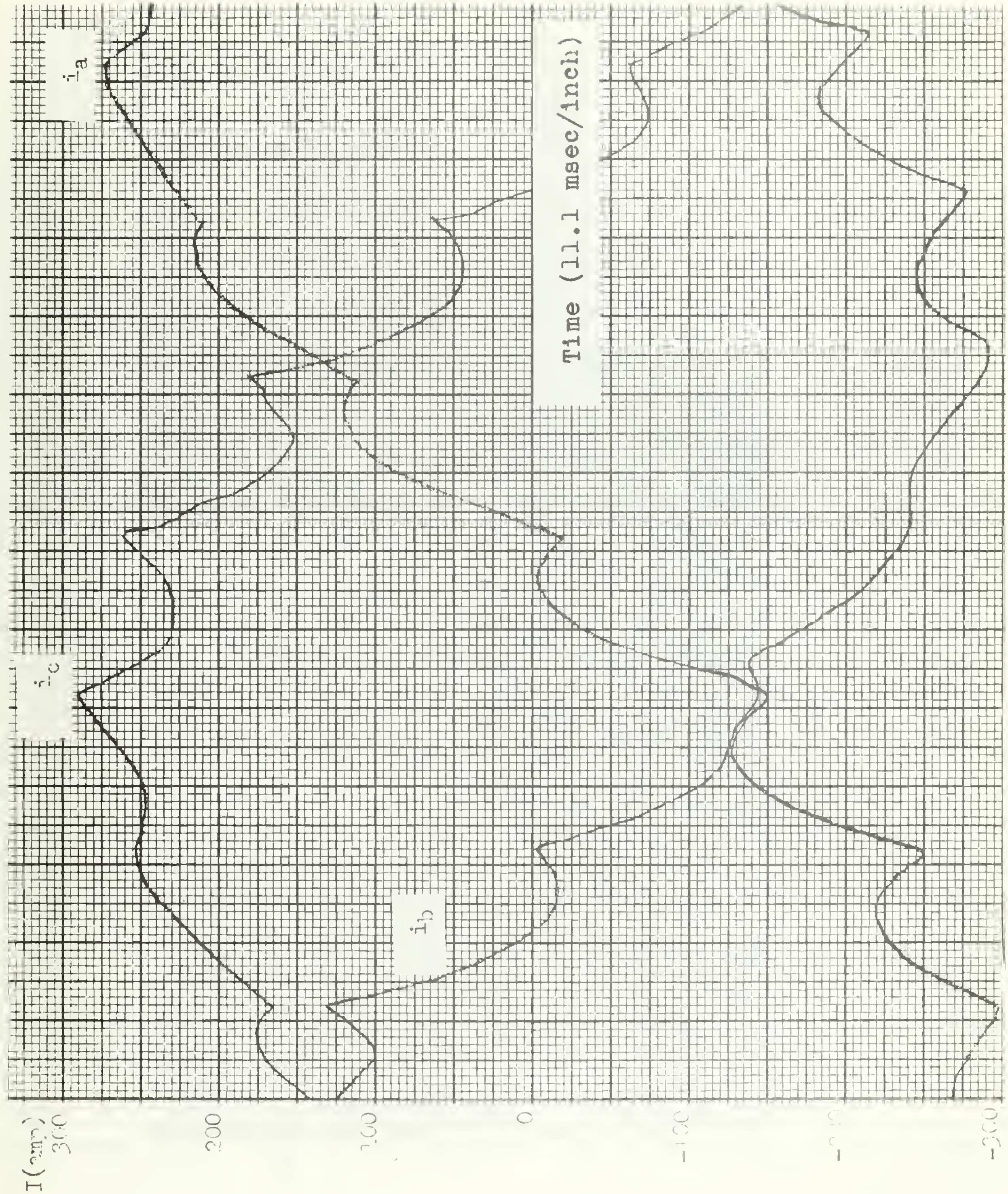
Generated Torque vs. Time. Constant-Frequency, Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-10



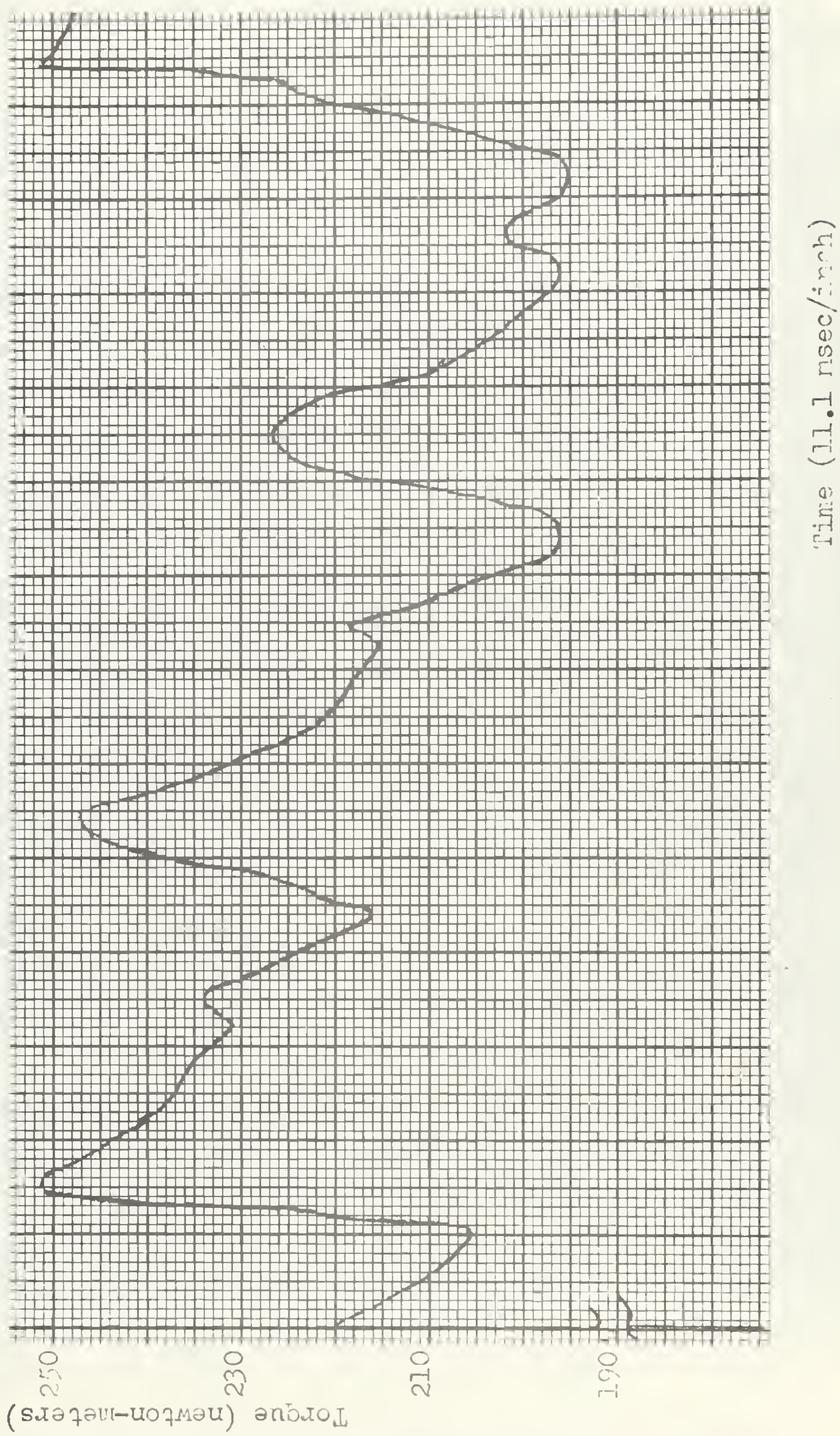
Stator Phase Currents vs. Time. Non-Sinusoidal Voltage Supply With Frequency 90 Radians Per Second Greater Than Rotor Angular Velocity.

Figure 6-11



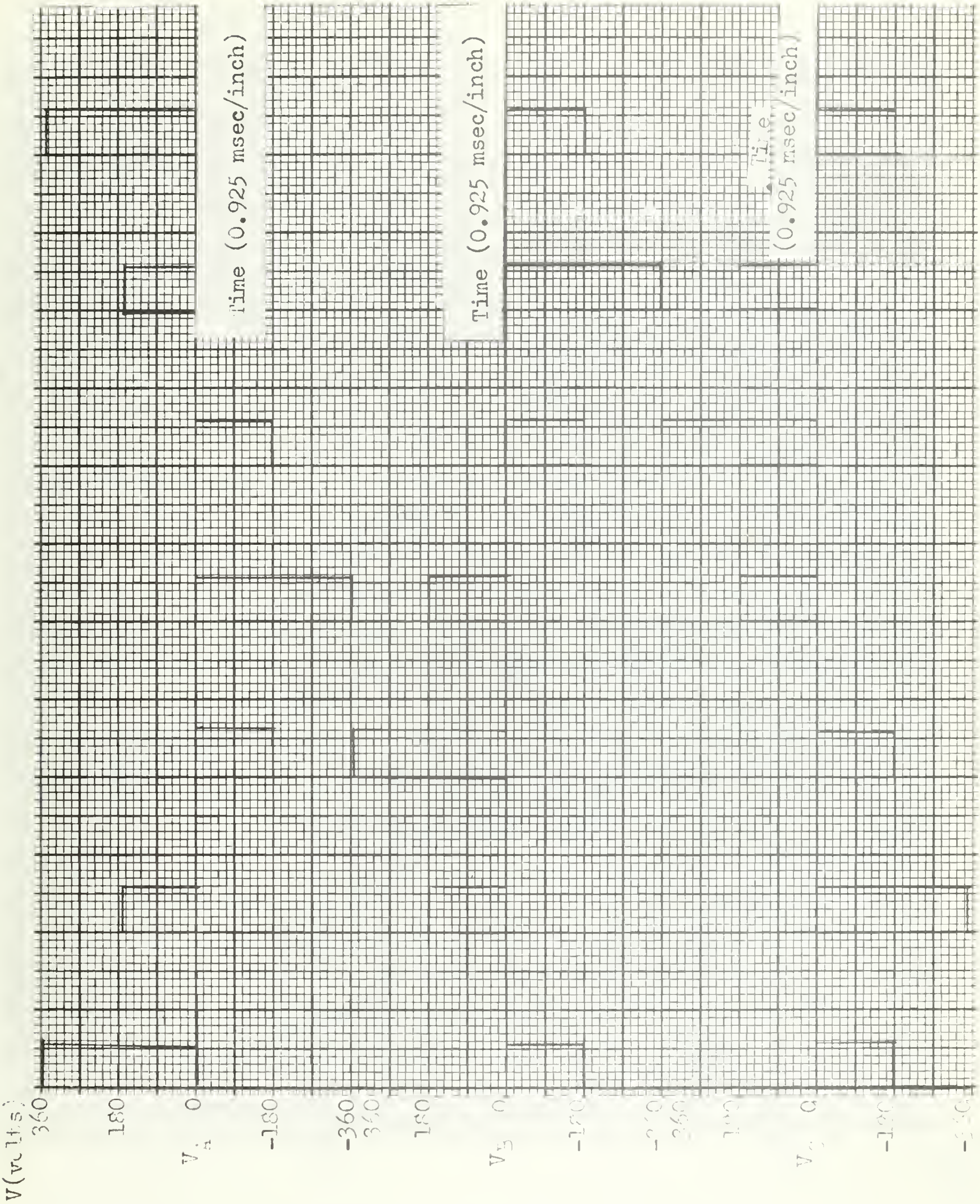
Rotor Phase Currents vs. Time. Non-Sinusoidal Voltage Supply With Frequency 90 Radians Per Second Greater Than Rotor Angular Velocity. Motor Undergoing Starting Transients.

Figure 6-12



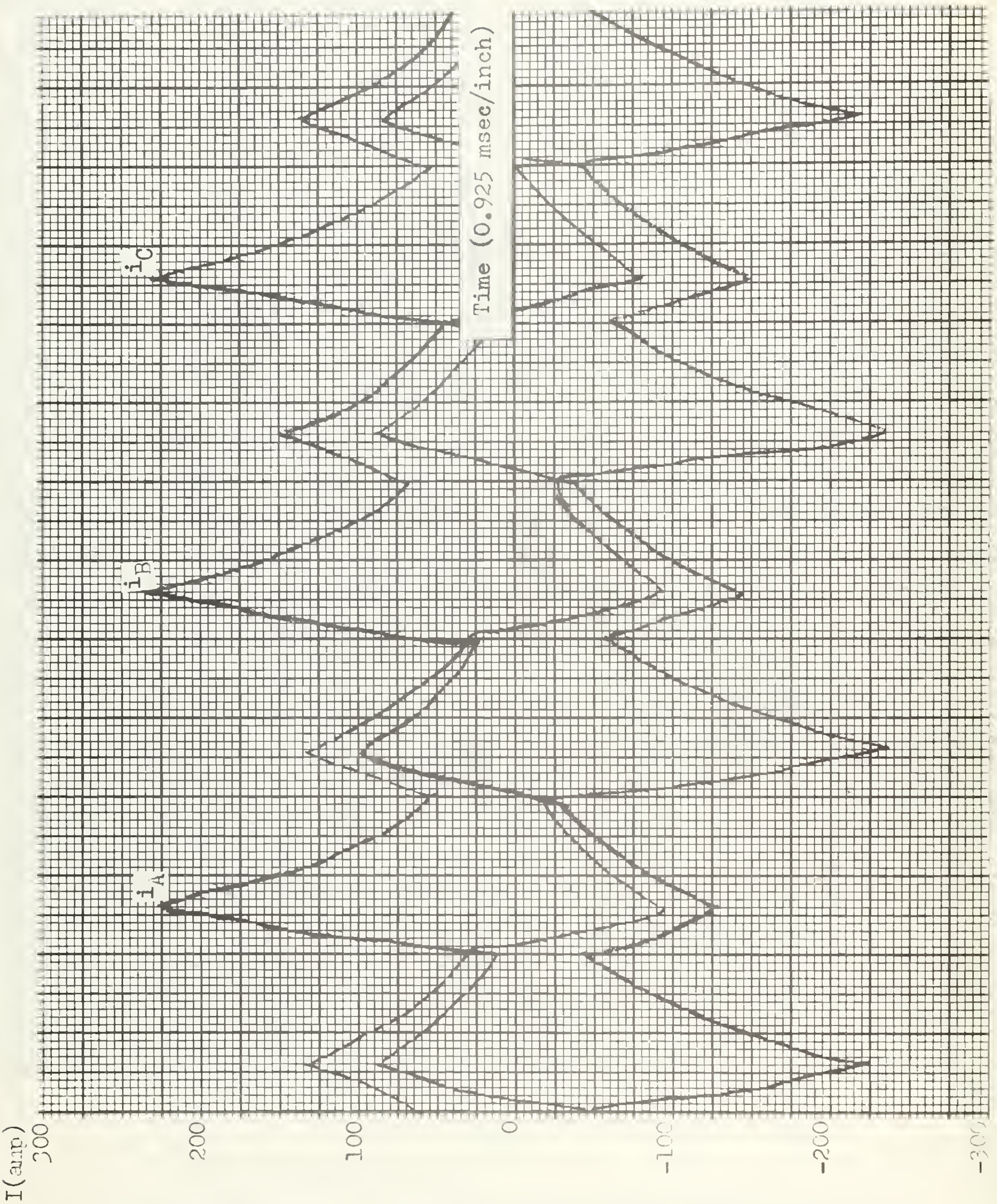
Generated Torque vs. Time. Non-Sinusoidal Voltage Supply With Frequency 90 Radians Per Second Greater Than Rotor Angular Velocity. Motor Undergoing Starting Transients.

Figure 6-13



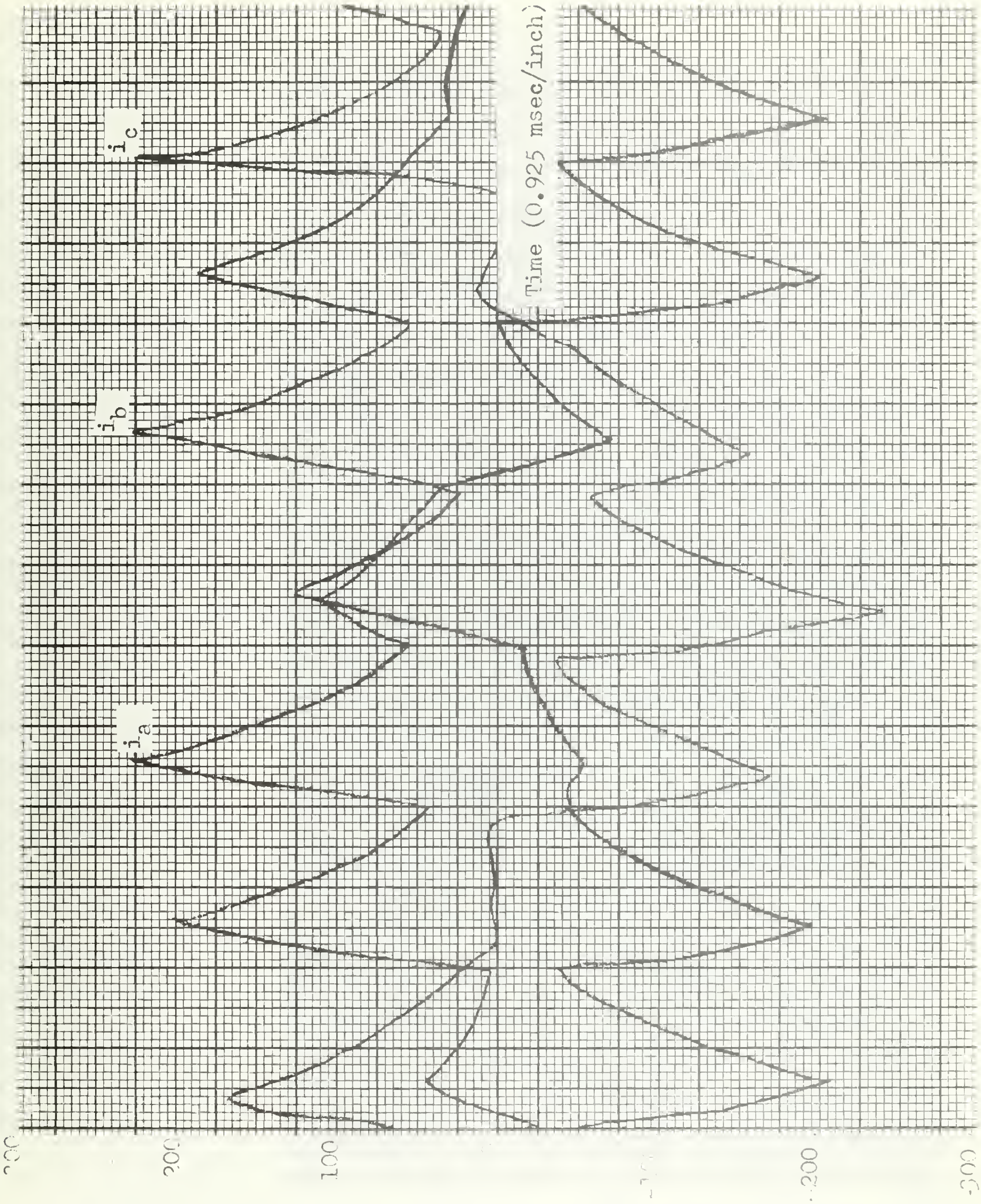
Stator Phase Voltages vs. Time. Variable Frequency, Variable Pulse-Width Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-14



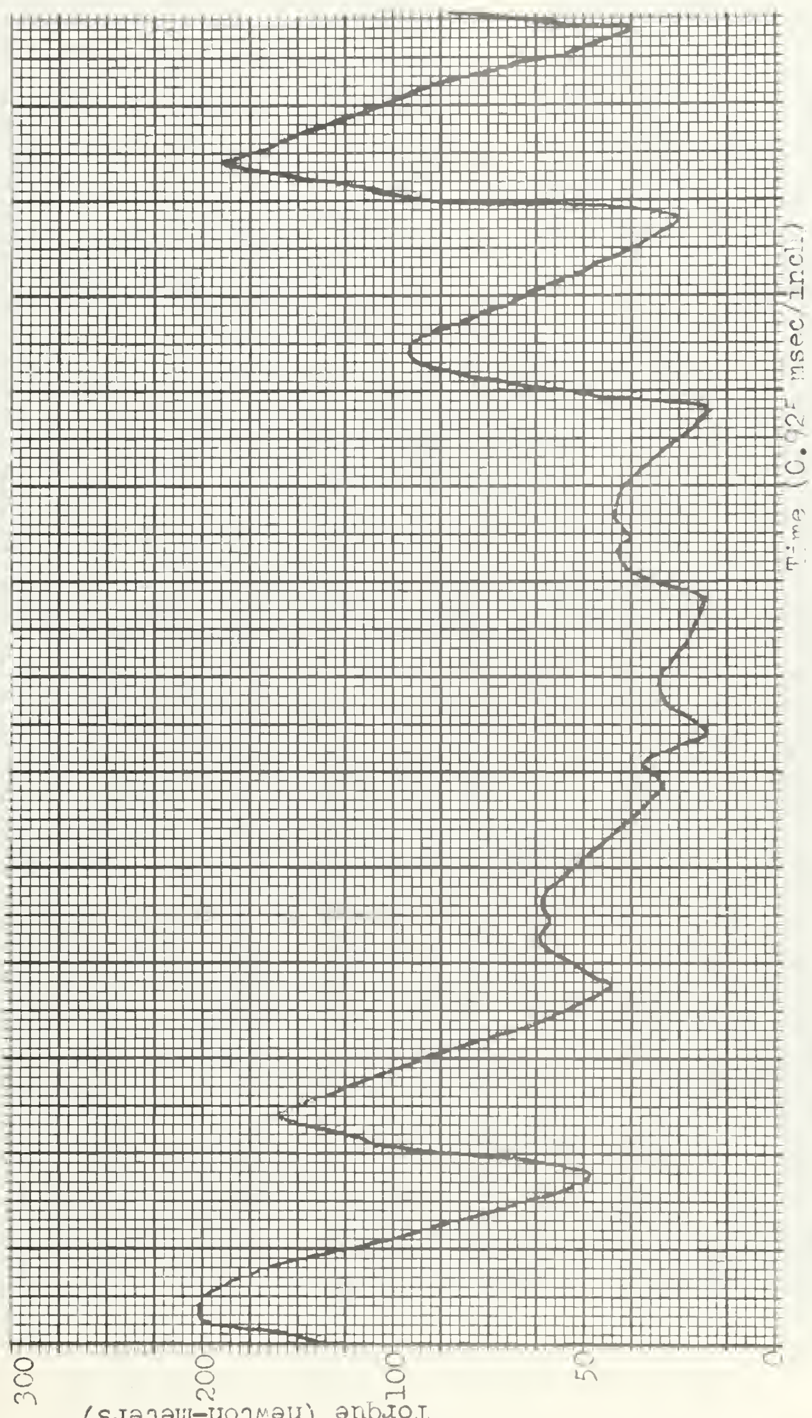
Stator Phase Currents vs. Time. Variable Frequency, Variable Pulse-Width Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-15



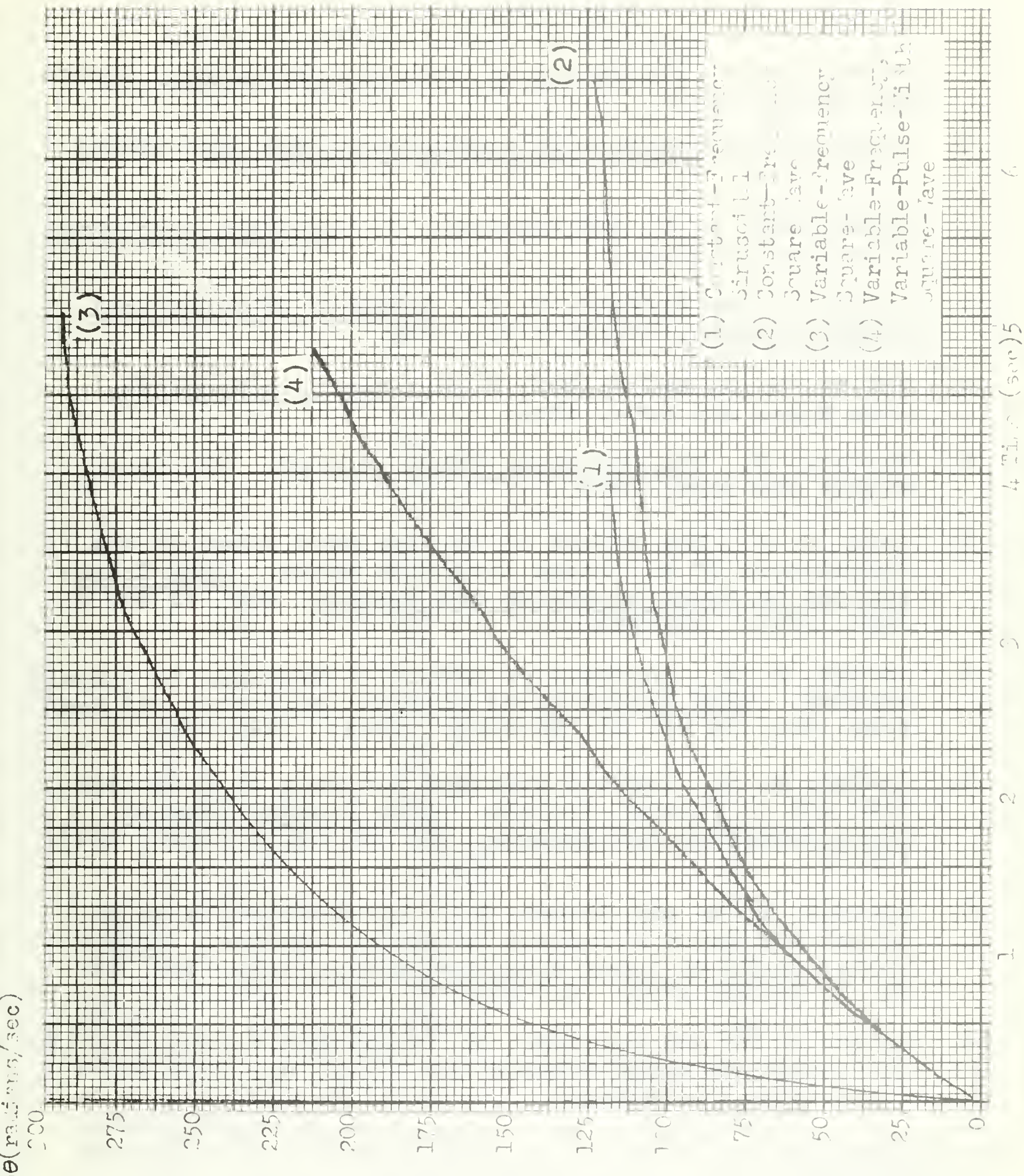
Rotor Phase Currents vs. Time. Variable Frequency, Variable Pulse-Width Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-16



Generated Torque vs. Time. Variable Frequency, Variable Pulse-Width Non-Sinusoidal Voltage Supply. Motor Undergoing Starting Transients.

Figure 6-17



Rotor Angular Velocity vs. Time. Comparison of
Motor Response to Different Voltage Inputs.

Figure 6-18

BIBLIOGRAPHY

1. Alger, P. L., The Nature of Induction Machines. Gordon and Beach, 1965.

2. Dawes, C. L., Industrial Electricity, 2nd ed. McGraw-Hill, 1942.

3. Fitzgerald, A. E. and Kingsley, C., Electric Machinery, McGraw-Hill, 1952.

4. Still, A. and Siskind, C. S., Elements of Electrical Machine Design, McGraw-Hill, 1954.

5. Thaler, G. J. and Wilcox, M. L., Electric Machines, John Wiley and Sons, 1966.

6. Vickers, H., The Induction Motor, 2nd ed. Sir Isaac Pitman and Sons, 1953.

7. Jain, G. C., The Effect of Voltage Wave Shape on the Performance of a 3-Phase Induction Motor. IEEE Transactions on Power Apparatus and Systems, Vol. 83 No. 6 June, 1964: 561-566.

8. Jordan, H. E., Analysis of Induction Machines in Dynamic Systems. IEEE Transactions on Power Apparatus and Systems, Vol. 84 No. 11, November, 1965: 1080-1088.

9. Jordan, H. E., Digital Computer Analysis of Induction Machines in Dynamic Systems. IEEE Transactions on Power Apparatus and Systems, Vol. 86 No. 6, June, 1967: 722-727.

10. Sabbagh, E. M. and Shewan, W., Characteristics of an Adjustable Speed Polyphase Induction Machine. IEEE Transactions on Power Apparatus and Systems, Vol. 87 No. 3, March, 1968: 613-623.

11. Smith, I. R. and Sriharan, S., Transient Performance of the Induction Motor Proceedings. IEE, Vol. 113 No. 7, July, 1966: 1173-1181.

APPENDIX I

FORTRAN IV COMPUTER PROGRAM FOR SIMULATION OF THREE PHASE INDUCTION MOTOR PERFORMANCE WHEN OPERATING ON THREE-PHASE, VARIABLE PULSE-WIDTH, VARIABLE FREQUENCY, SQUARE WAVE VOLTAGE SUPPLY:

```
C     RA = ROTOR ANGLE RAD
C     W = ROTOR VEL  RAD/SCC
C     FT = INERTIA OF SYSTEM
C     TLK = LOAD TORQUE CONSTANT
C     F = FRICTION AND WINDAGE CONSTANT
C     T = TIME FROM START OF CYCLE
C     TS = TIME FROM START OF PROGRAM
C     L11 = STATOR PHASE INDUCTANCE
C     L1X = STATOR - ROTOR MUTUAL INDUCTANCE MAX
C     LXX = ROTOR PHASE INDICTANCE
C     RSP = RESISTANCE STATOR PHASE
C     RRP = RESTISTANCE ROTOR PHASE
C     CT = CYCLE TIME
C     PV = VOLTAGE PULSE LENGTH
C     DT = TIME INCREMENT
      IMPLICIT REAL*8(A-H,L,P-Z)
      DIMENSION R(6,6),V(6),Z(6,6),PA(6),B(6,6),C(6,6), AB(6),PAB(6),
     1 E(6),A1(200),A2(200),A3(200),A4(200),A5(200),A6(200),
     2A(6),V1(200),V2(200),V3(200),
     3 TOG(200),TLR(200),WW(200),TSG(200),AC(6),U(6,6),Y(6),S(6)
     4 , FFF(6), FF(6), RAA(200)
      RA =0.
      W = 0.0
      FT = 1.0
      TLK = 0.000315
      F = 0.0282
      T=0.
      L11 = 0.04987
      L1X = 0.0482
      LXX = 0.04987
      RSP = .3
      RRP= .36
      PII = 3.1416
      FREQ = (W + 90.0)/(2.0*PII)
      MMM = 10 -FREQ*10.0/60.
      CT = 1.0/FREQ
      DT = CT/60.0
      A(1)=0.
      A(2)=0.
      A(3)=0.
      A(4)=0.
      A(5)=0.
      A(6)=0.
      MM=0
      TS=0.
      VMAX = 540.
```

```
      VSM=VMAX/3.
      VLR=VMAX-VSM
      CCC = 0.0
      DO 90 I=1,6
      FF(I) = 0.0
   90 FFF(I)= 0.0
      ABC = 0.0
   70 ABC = ABC + 1.
      DO 20 NN=1,200
       MM = MM+1
      FREQ=(W + 90.0)/(2.*PII)
      CT= 1./FREQ
      PV = CT/6.
      DT = PV/10.
      CCC = CCC + 1.2
          XXX = FREQ*10.0/60.
      MMM = XXX
      YYY = XXX - MMM
      IF(YYY .LT. 0.5)GO TO 41
      MMM = MMM + 1
   41 CONTINUE
      MMM = 10 - MMM
      AA1=RA
      AA2 = RA + 120.*PII/180.
      AA3 = RA - 120.*PII/180.
      R(1,1) = RSP
      R(1,2) = 0.
      R(1,3) = 0.
      R(1,4)= -W*L1X*DSIN(AA1)
      R(1,5)= -W*L1X*DSIN(AA2)
      R(1,6)= -W*L1X*DSIN(AA3)
      R(2,1) = 0.
      R(2,2) = R(1,1)
      R(2,3) =0.
      R(2,4) = R(1,6)
      R(2,5) = R(1,4)
      R(2,6) = R(1,5)
      R(3,1) = 0.
      R(3,2) = 0.
      R(3,3) = R(1,1)
      R(3,4) = R(1,5)
      R(3,5) = R(1,6)
      R(3,6) = R(1,4)
      R(4,1)= W*L1X*DSIN(-RA)
      R(4,2)= W*L1X*DSIN(2.*PII/3. -RA)
      R(4,3) = W*L1X*DSIN(-2.*PII/3. -RA)
      R(4,4) = RRP
      R(4,5) = 0.
```

```
R(4,6) = 0.
R(5,1) = R(4,3)
R(5,2)= R(4,1)
R(5,3)= R(4,2)
R(5,4) = 0.
R(5,5) = RRP
R(5,6) =0.
R(6,1)= R(4,2)
R(6,2) = R(5,1)
R(6,3) = R(5,2)
R(6,4) = 0.
R(6,5) = 0.
R(6,6) = RRP
Z(1,1) = L11
Z(1,2)= L1X*DCOS(2.*PII/3.)
Z(1,3)= L1X*DCOS(-2.*PII/3.)
Z(1,4)= L1X*DCOS(AA1)
Z(1,5)= L1X*DCOS(AA2)
Z(1,6) = L1X*DCOS(AA3)
Z(2,1)= Z(1,3)
Z(2,2) = L11
Z(2,3) = Z(1,2)
Z(2,4) = Z(1,6)
Z(2,5) = Z(1,4)
Z(2,6) = Z(1,5)
Z(3,1)= Z(1,2)
Z(3,2)= Z(1,3)
Z(3,3) = Z(1,1)
Z(3,4) = Z(1,5)
Z(3,5) = Z(2,4)
Z(3,6) = Z(1,4)
Z(4,1)= L1X*DCOS (-RA)
Z(4,2) = L1X*DCOS(2.*PII/3. -RA)
Z(4,3)= L1X*DCOS(-AA2)
Z(4,4) = LXX
Z(4,5)= Z(1,2)
Z(4,6)= Z(1,3)
Z(5,1) =  Z(4,3)
Z(5,2)= Z(4,1)
Z(5,3) = Z(4,2)
Z(5,4)= Z(1,3)
Z(5,5) =  LXX
Z(5,6)= Z(1,2)
Z(6,1) = Z(4,2)
Z(6,2)= Z(5,1)
Z(6,3)= Z(5,2)
Z(6,4)= Z(1,2)
```

```
      Z(6,5)= Z(1,3)
      Z(6,6) = LXX
      CALL GAUSS3(6,0.0001,Z,U,KK,6)
   27 IF(MM .LE. 10-MMM)GO TO 21
      IF(MM .LE. 10)GO TO 31
      IF(MM .LE. 20-MMM)GO TO 22
      IF(MM .LE. 20)GO TO 31
      IF(MM .LE. 30-MMM)GO TO 23
      IF(MM .LE. 30)GO TO 31
      IF(MM .LE. 40 - MMM)GO TO 24
      IF(MM .LE. 40) GO TO 31
      IF(MM .LE. 50 - MMM)GO TO 25
      IF(MM .LE. 50)GO TO 31
      IF(MM .LE. 60-MMM)GO TO 26
      IF(MM .LE. 60)GO TO 31
      MM = 1
      GO TO 27
   21 V(1)= VLR
      V(2)=-VSM
      V(3)=-VSM
      GO TO 30
   22 V(1)= VSM
      V(2)=VSM
      V(3)=-VLR
      GO TO 30
   23 V(1)= -VSM
      V(2) = VLR
      V(3)=-VSM
      GO TO 30
   24 V(1) = -VLR
      V(2)=VSM
      V(3)=VSM
      GO TO 30
   25 V(1)= -VSM
      V(2)=-VSM
      V(3)=VLR
      GO TO 30
   26 V(1) = VSM
      V(2) = - VLR
      V(3)= VSM
      GO TO 30
   31 V(1)= 0.0
      V(2)= 0.0
      V(3)= 0.0
   30 V(4) =0.
      V(5) =0.
      V(6) =0.
```

```
      T = T + DT
      TS = TS + DT
      DO 101 II=1,6
      Y(II)= 0.0
      DO 101 N=1,6
      PP = DABS(U(II,N))
      QQ=DABS(V(N))
      IF(PP .LT.0.000001 .OR. QQ.LT .0.000001) GO TO 101
      Y(II)= Y(II) + U(II,N)*V(N)
101    CONTINUE
      DO 102 II=1,6
      DO 102 M=1,6
      C(II,M)=0.0
      DO 102 N=1,6
      PP=DABS(U(II,N))
      QQ = DABS(R(N,M))
      IF(PP .LT. 0.000001 .OR. QQ .LT. 0.000001) GO TO 102
      C(II,M)= C(II,M) + U(II,N)*R(N,M)
102   CONTINUE
108   CONTINUE
      DO 103 II=1,6
      S(II) = 0.0
      DO 103 N= 1,6
      PP=DABS(C(II,N))
      QQ=DABS(A(N))
      IF(PP .LT. 0.000001 .OR. QQ .LT. 0.000001) GO TO 103
      S(II) = S(II) + C(II,N)*A(N)
103   CONTINUE
      DO 104 I = 1,6
104    PA(I) = Y(I) -S(I)
      DO 3 K=1,6
    3 AB(K)= A(K) + DT * PA(K)
      DO 105 II=1,6
      Y(II) = 0.0
      DO 105 N=1,6
      PP=DABS(U(II,N))
      QQ=DABS(V(N))
      IF(PP .LT. 0.000001 .OR. QQ .LT. 0.000001) GO TO 105
      Y(II) = Y(II) + U(II,N)*V(N)
105   CONTINUE
    1 DO 106 II=1,6
      S(II) = 0.0
      DO 106 N=1,6
      PP=DABS(C(II,N))
      QQ=DABS(AB(N))
      IF(PP.LT. 0.000001 .OR. QQ .LT. 0.000001) GO TO 106
      S(II) = S(II) + C(II,N)*AB(N)
106   CONTINUE
```

```
      DO 107 I=1,6
107    PAB(I) = Y(I) - S(I)
      DO 5  I=1,6
    5 AC(I) = A(I) + DT * (PA(I) + PAB(I))/2.
      DO 6 I=1,6
      E(I)=0.0
      PP = DABS(AC(I))
      IF(PP .LT. 0.000001) GO TO 8
      E(I)=(AC(I) -AB(I))/AC(I)
      IF( E(I).LE. 0.0) GO TO 7
    9 IF( E(I).LE. 0.05) GO TO 8
      GO TO 10
    7 E(I)= -E(I)
      GO TO 9
    8 CONTINUE
    6 CONTINUE
      F1 = DSIN(AA1)
      F2 = DSIN(AA2)
      F3 = DSIN(AA3)
      TQ = -AC(1)*L1X*(AC(4)*F1+AC(5)*F2+AC(6)*F3) -AC(2)*L1X*
     1  (AC(4)*F3+AC(5)*F1+AC(6)*F2) -AC(3)*L1X*(AC(4)*
     2 F2+AC(5)*F3+AC(6)*F1)
      WDOT = (TQ - TLK*(W**2) - F*W)/FT
      W = W + DT*WDOT
      RA = RA + W*DT
      DO 12 I=1,6
      A(I)=AC(I)
   12 AB(I) = AC(I)
      WW(NN) = W
      TQG(NN) = TQ
      TLR(NN) = TLK*(W**2)
      V1(NN) = V(1)
      V2(NN)= V(2)
      V3(NN)= V(3)
      A1(NN)= A(1)
      A2(NN)= A(2)
      A3(NN)= A(3)
      A4(NN)= A(4)
      A5(NN)= A(5)
      A6(NN)= A(6)
      TSG(NN) = TS
      RAA(NN) = RA
      GO TO 11
   10 DO 50 I=1,6
   50 AB(I) = AC(I)
      GO TO 1
   11 CONTINUE
   20 CONTINUE
```

```
      DO 40 NN=1,200
      WRITE(6,100)A1(NN),A2(NN),A3(NN),A4(NN),A5(NN),A6(NN), TOG(NN),
     1 WW(NN),TSG(NN),RAA(NN),NN
  100 FORMAT(8F8.2,2F10.3,I6)
   40 CONTINUE
      IF(ABC .LT. 50.)GO TO 70
      STOP
      END
```

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library Naval Postgraduate School Monterey, California 93940	2
3.	Naval Ship System Command (Code 2052) Navy Department Washington, D. C. 20360	1
4.	Prof. Milton L. Wilcox Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1
5.	Electrical Engineering Department Naval Postgraduate School Monterey, California 93940	1
6.	Lt. R. P. Wells, USN Box 34 Ship Repair Facility Subic, P.I. FPO San Francisco 96662	1

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

INVESTIGATION AND ANALYSIS OF THREE-PHASE INDUCTION MOTOR PERFORMANCE WITH NON-SINUSOIDAL VOLTAGE SUPPLY

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Thesis

5. AUTHOR(S) *(First name, middle initial, last name)*

WELLS, Richard Paul, Lieutenant, USN

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1968	50	11

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT [redacted]

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

The development of vehicles powered by direct-current sources together with the development of the silicon-controlled rectifier has led to the use of squirrel-cage induction motors operating on non-sinusoidal, variable-frequency voltage supplies. A digital computer simulation of the transient and steady-state performance of a three-phase motor is derived and its use in predicting motor operation is illustrated.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

UNCLASSIFIED

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
VARIABLE SPEED INDUCTION MOTOR						
PULSE DRIVEN INDUCTION MOTOR						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0101-807-6821

A-31409

[redacted]